Apply fuzzy rough sets to evaluate the inter-class similarity

1 fuzzy sets

Fuzzy sets play an important role in addressing the uncertainty of information.

The definition of fuzzy sets is given by Zadeh[1]

Definition1 Given a universe of discourse U, a fuzzy set A is a set function on U into [0,1], that is , where is the membership function of A and .

As the same with conventional set theory, the fuzzy sets also have union, intersection, and complement operators. However, the fuzzy sets operators are not restricted strictly in a kind of form. When an operator satisfies some essential conditions, we can say this one can be appropriately used in fuzzy sets. It means that fuzzy union, fuzzy intersection and fuzzy complement have more than one formula.

Fuzzy union is defined as s-norm, that is , where and have the following attributes:

1): (boundary condition)

2):(commutativity)

3):if and then (monotonicity)

4): (associativity)

Fuzzy intersection is given as T-norm, , that is , which also satisfies the commutativity, monotonicity, associativity like s-norm. The only difference is that the boundary condition of t-norm is .

Furthermore, given a s-norm, we can define another operator , that is . Correspondingly, based on t-norm, the binary operator is . The four operators s-norm, t-norm, and have the following properties, , ,,,where N is the fuzzy complement and the standard formula is .

Fuzzy intersection is given as t-norm, that is where and satisfies the following conditions:

1.  (boundary condition)
2. (commutativity)
3. if and then (monotonicity)
4. (associativity)

A fuzzy equivalence relation is a function and satisfy the following conditions:

1. (reflexivity)
2. (symmetry)
3. (min-max-transitivity)

Especially, when the operator ‘min’ is replaced by t-norm, that is  , we call the D is a fuzzy T-equivalence relation.

[10] gives some widely used functions which satisfy the above conditions:

2 rough sets

Rough sets [2] are particularly useful in dealing with ambiguity, vagueness and general uncertainty problems.

Let be an information system, where U is a nonempty and finite set of objects, C is a nonempty and finite set of attributes. Given an equivalence relation R, the universe U is divided into a family of disjoint sunsets, named equivalence class . For ,

the equivalence relation R is satisfied the following rules:



According to [], given a subset of objects and an equivalence relation R, the lower and upper approximations of X are defined as:



When , we say X is a rough set in the approximation space. Obviously, the subset X is approximated by two unions of equivalent classes. The lower approximation of X is represented by the union of equivalence classeswhich are totally contained by X. The upper approximation of X is evaluated by the union of equivalence classeswhich have non-empty intersection with X. The difference between and is the boundary region.

3 the combination fuzzy sets with rough sets

Although the classical rough sets proposed by have been applied in many fields and also have obtained outstanding performance, there is a restriction that we couldn’t ignored.

That is, the classical rough sets are considerably effective only when the data is symbolic-valued. To address the above restriction and broaden the application range of rough sets, [3 ] Didier et.al imposed a novel theory named fuzzy rough sets which combined fuzzy sets with rough sets. The key of the fuzzy rough sets is to supersede the equivalence relation of rough sets by a fuzzy equivalence relation.

Definition: Given a universe of discourse U, R is a fuzzy equivalence relation on U. For , we have



where X is a subset of U. Based on [4][5], If the R is a fuzzy T-equivalence relation on U, the operator t-norm, s-norm, and  defined above can be applied to substitute ‘max’-‘min’, that is



and



Where X is the subset of U and N is the fuzzy complement. , and are the lower approximation and mean the degrees the x certainly belongs to the set X. , and are the upper approximation and denote the degrees the x possibly belongs to set X.

4 evaluate the similarity between two sets

Given a finite and nonempty set of samples U, a feature set A, and decision D which partitions the samples into subsets. For , if ,, otherwise, . Inspired by [2], the lower approximation of fuzzy rough sets can be utilized to evaluate the membership degree of x to the th class. Here we choose and to compute the lower approximation.





Where is the fuzzy similarity relation between samples and can be computed by some kernel functions[6]:

1. Gaussian kernel: 
2. Rational quadratic kernel: 

and represent that the degree of x to  rely on the closest sample which belongs to another class. Considering a special situation, there are only two classes A and B. Theanddenote that the farther a sample x and class B are, the greater degree x belongs to class A. Therefore, the similarity between class A and class B can be evaluated by computing the fuzzy lower approximation of all samples in class A.

Assuming class A has m elements and class B has k elements, the fuzzy similarity between class A and class B can be defined as



where , .

What we must pay attention is that , since . To make the fuzzy similarity satisfy the symmetric condition, the dual fuzzy similarity can be introduced as

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The is a novel approach which mix up fuzzy sets and rough sets can be employed to compute the inter-class similarity.

Literature

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